Structure stability of ultraintense laser pulse in transverse homogeneous cold plasma

Hai. Lin*
Graduate School, China Academy of Engineering Physics, P.O. Box 2101, Beijing 100088, China

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We study transverse structure symmetry of an ultraintense laser pulse through transverse homogeneous cold plasma. We derive a steady-structure equation of laser pulse and solve it under different on-axis conditions. We compare Hamiltonian values at solutions with different on-axis conditions to examine their relative stability. Numerical results show that for different ionic density, symmetric structure is not always stable relative to asymmetric one of same power. For a given ionic density, whether a symmetric structure is stable is determined by its power. This result agrees with the phenomenon of pulse ''head bending'', qualitatively. Our theory reveals that, in addition to the plasma's transverse inhomogeneity, there is another mechanism responsible for asymmetric structure.

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The progress in compact terrawatt laser [1] enables ultrarelativistic pulse of intensity $10^{18\sim 19} W/\mathrm{cm}^2$ available in various applications. High toroidal dc magnetic field due to hot electrons has been found in some experiments and computer simulations [2]. For an ultrarelativistic laser with dimensionless field amplitude $eA/m_{e,0}c^2 > 1$, direct longitudinal $v_e \times B$ acceleration of electrons is the main mechanism of hot electron generation in contrast to other mechanisms, for instance, Brunel vacuum heating [3], and wave breaking of laser wakefield [4].

For inertial confinement fusion application, the spatial symmetry of pressurization on an overdense target core by hot electrons is a crucial issue. If in underdense plasma the direction of Lorentz force deviates from laser propagation axis (z axis), generated hot electrons will also follow this deviation and thus pressurize the target core asymmetrically. Since $v_e \times B$ Lorentz force has an equivalent form ∇A^2, ponderomotive exerted by pulse structure, the transverse structure symmetry of pulse in underdense plasmas is crucial to ensure symmetric pressurization on an overdense core. In some computer simulations [5], an interesting phenomenon that the leading edge of the pulse is bent near critical surface but the pulse body remains straight, was found. Moreover, there have been experimental [6] and theoretic works [7] on deflection of a laser beam by a transverse plasma current. It is easy to understand this latter beam deflection since it occurs in transverse inhomogeneous plasma that is produced by asymmetrically arranged preforming beams. In contrast, whether a pulse can spontaneously lose its transverse symmetry in homogeneous plasmas is a more attractive problem. If we only ascribe beam bending to plasma transverse inhomogeneity, any part of the pulse will be bent by this inhomogeneity, which is not consistent with results in Ref. [5].

In this paper, we attempt to explain the phenomenon in Ref. [5]. For an ultraintense laser, its structure in homogeneous plasma is complex due to nonlinear optical effect. Even though plasma is homogeneous, sufficiently high laser intensity, as revealed in Ref. [8], will lead to self-focusing structure that is inhomogeneous but axisymmetric. Here, we focus our attention to nonaxisymmetric self-focusing structure. On the basis of a Lagrangian and Hamiltonian of a radiation field, we discuss the possibility of nonaxisymmetric structure of an ultraintense laser in transverse homogeneous plasma. We derive an equation of pulse structure and seek its different solutions with respective space symmetry. For these solutions, we develop a Hamiltonian approach to examine their relative stability. Our numerical results reveal that in some parameter region, a transverse symmetric pulse structure becomes unstable relative to an asymmetric one.

The interaction of an ultraintense laser with plasma has two important nonlinear features: ponderomotive cavitation and relativistic correction of electron mass. Here, ponderomotive cavitation refers to the decrease of electron density since laser field expels electrons away. These two features are contained in the nonlinear *Schrödinger* equation of laser vector potential A [8]

$$i\partial_t a = -\frac{1}{2\omega}[c^2\nabla_\perp^2 a + V(a)]a;$$
$$V = \frac{\omega_{p,0}^2}{\gamma} - \omega^2 + \frac{c^2\nabla_\perp^2\gamma}{\gamma}; \tag{1}$$

where $a=\sqrt{I}\exp(i\theta)$ is the vector potential in pulse frame, γ is relativistic factor $\sqrt{1+I}$, c is light speed, and $\omega_{p,0}$ and ω are plasma frequency and laser frequency. The steady-state Lagrangian and Hamiltonian are

$$L^s(I,\nabla I,\mu) = \int \mu^* I d\tau - H^s,$$
$$H^s = \frac{1}{2\omega}\int\left[2\omega_{p,0}^2\sqrt{1+I} - \omega^2 I + \frac{(\nabla_\perp I)^2}{4I(1+I)}c^2\right]d\tau, \tag{2}$$

where $d\tau = dx\,dy\,d\xi$ is pulse volume, and $\mu = \partial_t\theta$ is space independent and can be taken as frequency-shift associated with spatial varying structure. Here, we have adopted $|x| \to \infty$ boundary condition $I(|x|\to\infty)=0$. A steady structure equation can be obtained from variational equation $\delta L^s/\delta I = 0$, i.e.,

*Email address: Hai_Lin72@hotmail.com

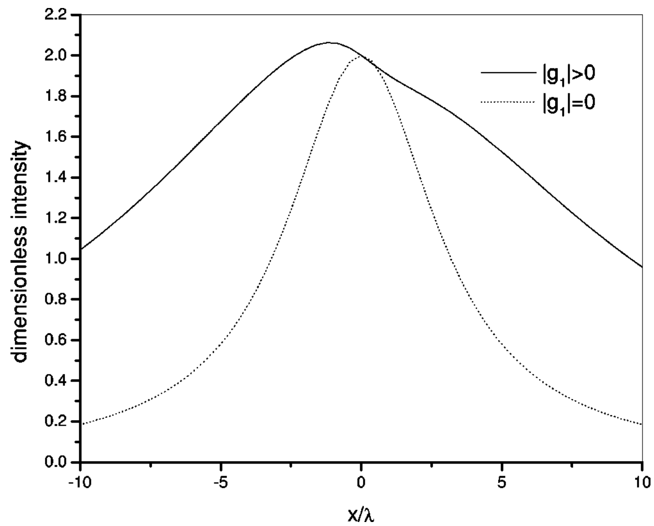

FIG. 1. Example of steady structures with fixed on-axis intensity.

$$[2\omega\mu+\omega^2]/c^2=-\frac{2}{4I(I+1)}\nabla^2 I+\frac{2I+1}{4I^2(I+1)^2}(\nabla I)^2+\frac{\omega_{p,0}^2}{c^2}\frac{1}{\sqrt{1+I}}. \tag{3}$$

It should be noted that this equation depends on ion density N_i via plasma frequency $\omega_{p,0}$.

For homogeneous plasmas of ion density N_i, the spatial symmetry of solution of Eq. (3) is determined by on-axis condition $\nabla I|_{|x|=0}$. Different solutions with respective on-axis conditions represent various possible pulse structures. Relative stability among those structures are related with their respective energies or Hamiltonian values. In principle, any solution with a respective on-axis condition should be solved from nonlinear Eq. (3). This is a cumbersome computational task. Moreover, for any pulse section, its power is given but its transverse structure has multiple possible forms with respective on-axis conditions. To specify the stablest structure, we compare Hamiltonian values of different structures. We stress that this comparison should be confined in structures with the same power. Otherwise, comparison among structures with different power, i.e., at different pulse section is not pertinent to consideration of stability of pulse structure. In order to save computer time, and to obtain a set of solutions corresponding to the same laser power, we develop an approximate method.

The essence of this approximation is the invariance of variational equation $\delta L^s/\delta I=0$:

$$\left.\frac{\delta L^s}{\delta I}\right|_{I=I_0+I_g}=0=\left.\frac{\delta L^s}{\delta I}\right|_{I=I_0},$$

We expand L^s around $a_0=\sqrt{I_0}e^{i\mu_0 t}$

$$L^s(a_g)=L^s(a_0)+dL^s,$$

$$dL^s=\int \mu_g I_0 d\tau+dH^s,$$

$$dH^s=\frac{1}{2\omega}\int [G_1 I_g^2+G_2(\nabla I_g)^2]d\tau,$$

where $a_g=\sqrt{I_0+I_g}e^{-i(\mu_0+\mu_g)t}$, $I_g=I_0 g$. I_0 is the even function of $x_\perp$ while g is the odd function of $x_\perp$. μ_g is space independent and can be treated as a frequency shift caused by I_g. Coefficients in integral kernel of dH are

$$G_1=-\frac{\omega_{p,0}^2}{2}(1+I_0)^{-3/2}+c^2\frac{\nabla^2 I_0}{2}\frac{2I_0+1}{(I_0^2+I_0)^2};$$

$$G_2=c^2\frac{1}{2}\frac{1}{I_0^2+I_0}.$$

Note that higher-order terms have been neglected because I_g is small, and odd-order expanding terms are absent since their spatial integrals are zero. The invariance of variational equation yields

$$0=\frac{\delta dL^s}{\delta I_0}+\frac{\delta dL^s}{\delta I_g}. \tag{4}$$

It should be stressed that $\delta dL^s/\delta I_0$ is the even function, while $\delta dL^s/\delta I_g$ is the odd one. Thus, Eq. (4) has a solution only when

$$\frac{\delta dL^s}{\delta I_0}=0,\qquad \frac{\delta dL^s}{\delta I_g}=0. \tag{5}$$

The second equality in Eq. (5) reads

$$\nabla^2 g+\left[-\frac{\nabla I_0}{1+I_0}\right]\nabla g+\left[\frac{\omega_{p,0}^2}{c^2}\frac{I_0}{\sqrt{1+I_0}}-\frac{\nabla^2 I_0}{1+I_0}-\left(\frac{\nabla I_0}{I_0}\right)^2\right]g=0, \tag{6}$$

and the first one in Eq. (5) gives the relation between μ_g and I_g:

$$\mu_g=\frac{dH^s}{\int I_0 d\tau}. \tag{7}$$

Obviously, $g(x)=0$ is a trivial solution of Eq. (6) and meets on-axis condition $\partial_\perp g|_{x=0}=0$, correspondingly, we obtain $dH^s|_{g(x)=0}=0$. Other nontrivial solutions meeting respective on-axis condition $\partial_\perp g|_{x=0}\neq 0$ can also be solved from Eq. (6). To examine their stability, one can compare the Hamiltonian increment dH^s or μ_g at those solutions with that at $g(x)=0$. If solutions with negative dH^s exist, the I_0 structure is unstable.

In the following numerical calculation, we put field vector potential a in units of Compton potential $m_{0,e}c^2/e$, length in units of laser wavelength in microns λ, $\omega_{p,0}^2$ in units of $c^2/\lambda^2=\omega^2$, and N_i in units of critical density $m_{0,e}\omega^2/4\pi e^2$. In particular, our calculation is toward strong field limit $I>1$.

We first solve Eq. (3) to obtain an even solution $I_0=\sum_{i=1}^{i=\infty}\nu_i x^{2i-2}$ and a general solution $I_0=\sum_{i=1}^{i=\infty}\nu_i x^{i-1}$.

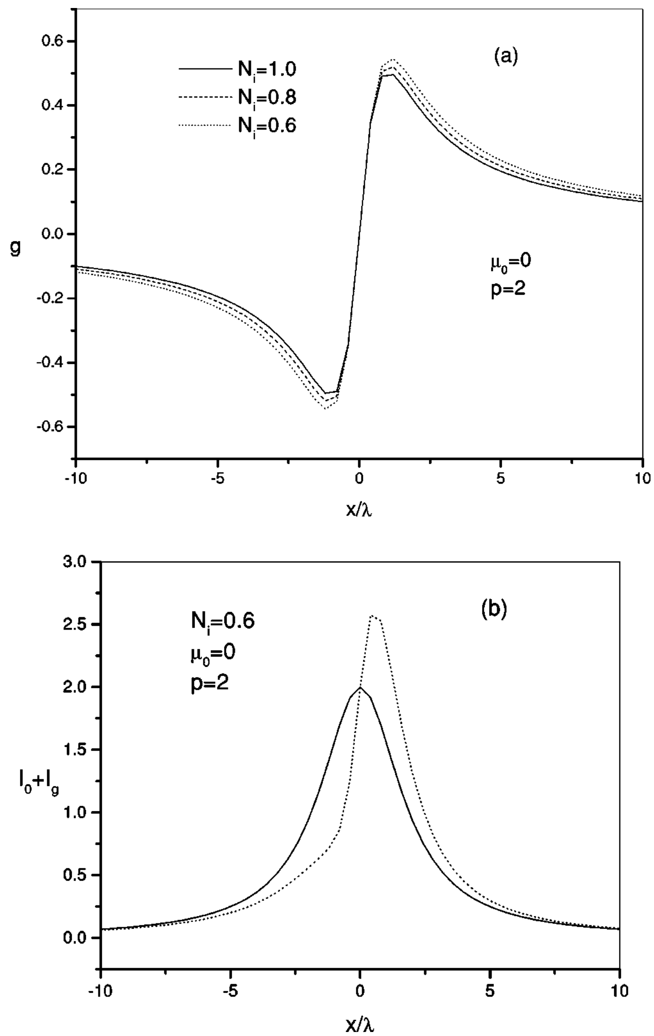

FIG. 2. (a) Example of g under different I_0 background; (b) relevant intensity profiles I_0+I_g.

These two solutions correspond to the same on-axis intensity $I_0(x=0)$. The results are plotted in Fig. 1 and indicate that these two solutions have different power $p=\int I dx dy$. Here, the existence of a general solution demonstrates solubility of nonlinear Eq. (3) under nonzero on-axis condition $\nabla I|_{x=0} \neq 0$. Then we present in Fig. 2 the nontrivial solutions of g at different I_0 background, and asymmetric structure $I=I_0+I_g$.

We present in Fig. 3 dH^s as the function of the power p for three values of N_i. It reveals in Fig. 3 that for the value of $N_i=0.8$, asymmetric distortion I_g with negative dH appear from some symmetric background at low power ($p<3$). For $N_i=1$, I_g with negative dH appear at a higher power ($p\sim5$). Therefore, we can see in Fig. 3 that for given plasma density, whether symmetric background support asymmetric distortion with negative dH is determined by its power.

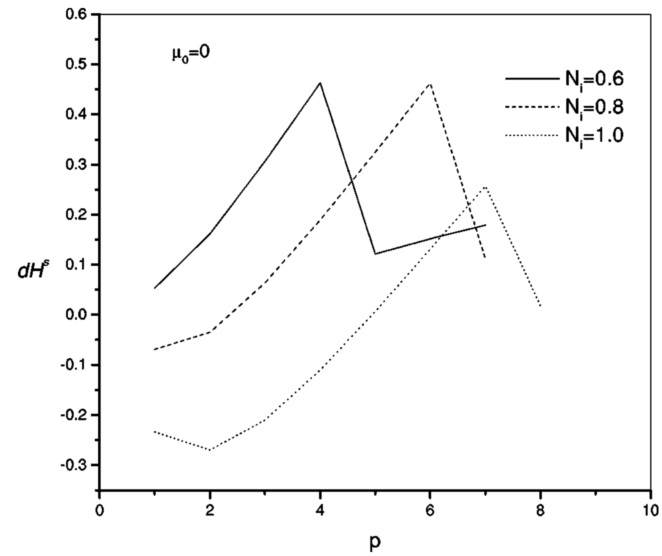

FIG. 3. Hamiltonian increment dH vs power p for different values of ion density N_i.

What is revealed in Fig. 3 agrees qualitatively with the simulation results in Ref. [5]. For real pulse structure, power at the leading edge is less than that at the pulse body. It is shown in Ref. [5] that when the pulse is in low-density plasmas, both its head (leading edge) and body remain straight, suggesting that symmetric transverse structures at the pulse head and at its body are both stable. This fact is reflected by the curve ($N_i=0.6$) in Fig. 3 that does not contain negative dH^s. With plasma density rising to critical density, Ref. [5] shows that the pulse head becomes bent whereas the pulse body remains straight. This fact is also reflected by the other two curves in Fig. 3. For these two curves, negative dH^s appear in the low power region while dH^s remain positive at the high power region.

We have investigated structure stability of the ultraintense laser pulse in transverse homogeneous plasmas via a Hamiltonian approach. Our numerical results indicate that the structure with lower Hamiltonian value is variant with laser power p and plasma density N_i. For some (p,N_i), an asymmetric steady structure corresponds to a lower Hamiltonian value than a symmetric one. The deformation we studied is spontaneous rather than induced by plasma inhomogeneity. With the help of this result, we explain the phenomenon presented in Ref. [5]. We attribute the head bending of the laser to the asymmetric structure of the lowest Hamiltonian in the situation of high plasmas density ($N_i\sim1$) and low power. This situation is fulfilled when the pulse head reaches critical surface.

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